Fluctuation cumulant behavior for the field-pulse-induced magnetization-reversal transition in Ising models

Arnab Chatterjee* and Bikas K. Chakrabarti[†]

Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India

(Received 22 October 2002; revised manuscript received 6 January 2003; published 21 April 2003)

The universality class of the dynamic magnetization-reversal transition, induced by a competing field pulse, in an Ising model on a square lattice, below its static ordering temperature, is studied here using Monte Carlo simulations. Fourth-order cumulant of the order parameter distribution is studied for different system sizes around the phase boundary region. The crossing point of the cumulant (for different system sizes) gives the transition point and the value of the cumulant at the transition point indicates the universality class of the transition. The cumulant value at the crossing point for low temperature and pulse width range is observed to be significantly less than that for the static transition in the same two-dimensional Ising model. The finite-size scaling behavior in this range also indicates a higher correlation length exponent value. For higher temperature and pulse width range, the transition seems to fall in a mean-field-like universality class.

I. INTRODUCTION

The response of a pure magnetic system to time-dependent external magnetic fields has been of current interest in statistical physics [1–3]. These studies, having close applications in recording and switching industry, have also got considerable practical importance. These spin systems, driven by time-dependent external magnetic fields, have basically got a competition between two time scales: the time period of the driving field and the relaxation time of the driven system. This gives rise to interesting nonequilibrium phenomena. Tóme and Oliveira first made a mean-field study [4] of kinetic Ising systems under oscillating field. The existence of the dynamic phase transition for such a system and its nature have been thoroughly studied using extensive Monte Carlo simulations. Later, investigations were extended to the dynamic response of (ferromagnetic) pure Ising systems under magnetic fields of finite-time duration [5]. All the studies with pulsed field were made below T_c^0, the static critical temperature (without any field), where the system gets ordered. A "positive" pulse is one which is applied along the direction of prevalent order, while the "negative" one is applied opposite to that. The results for the positive pulse case did not involve any new thermodynamic scale [5]. In the negative pulse case, however, interesting features were observed [5]: the negative field pulse competes with the existing order, and the system makes a transition from one ordered state characterized by an equilibrium magnetization $+m_0$ (say) to the other equivalent ordered state with equilibrium magnetization $-m_0$, depending on the temperature T, field strength h_p and its duration Δt. This transion is well studied in the limit $\Delta t \to \infty$ for any nonzero value of h_p at any $T < T_c^0$. This transition, for the general cases of finite Δt, is called here "magnetization-reversal" transition. Some aspects of this transition has been recently studied extensively [3,6].

II. MODEL AND THE TRANSITION

The model studied here is the Ising model with nearest-neighbor interaction under a time-dependent external magnetic field. This is described by the Hamiltonian,

$$H = -\frac{1}{2} \sum_{\{ij\}} J_{ij} S_i S_j - h(t) \sum_i S_i, \qquad (1)$$

where J_{ij} is the cooperative interaction between the spins at site i and j, respectively, and each nearest-neighbor pair is denoted by $\{\cdots\}$. We consider a square lattice. The static critical temperature is $T_c^0 = 2/\ln(1+\sqrt{2}) \simeq 2.269\ldots$ (in units of J/K_B). At $T < T_c^0$, an external field pulse is applied, after the system is brought to equilibrium characterized by an equilibrium magnetization $m_0(T)$. The spatially uniform field has a time dependence as follows:

$$h(t) = \begin{cases} -h_p, & t_0 \le t \le t_0 + \Delta t \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

Typical time-dependent (response) magnetization $m(t)$ $(=\langle S_i \rangle$, where $\langle \cdots \rangle$ denotes the thermodynamic "ensemble" average) of the system under different magnetic field $h(t)$ are indicated in the Fig. 1. The time t_0 at which the pulse is applied is chosen such that the system reaches its equilibrium at T $(<T_c^0)$. As soon as the field is applied, the magnetization $m(t)$ starts decreasing, continues until time $t + \Delta t$ when the field is withdrawn. The system relaxes eventually to one of the two equilibrium states (with magnetization $-m_0$ or $+m_0$). At a particular temperature T, for appropriate combinations of h_p and Δt, a magnetization-reversal transition occurs, when the magnetization of the system switches from one state of equilibrium magnetization m_0 to the other with magnetization $-m_0$. This reversal phenomena at $T < T_c^0$ is simple and well studied for $\Delta t \to \infty$ for any nonzero h_p. We study here the dynamics for finite Δt values. It appears that generally $h_p \to \infty$ as $\Delta t \to 0$ and $h_p \to 0$ as $\Delta t \to \infty$ for any such dynamic magnetization-reversal transition phase boundary at any temperature T $(<T_c^0)$. In

*Email address: arnab@cmp.saha.ernet.in
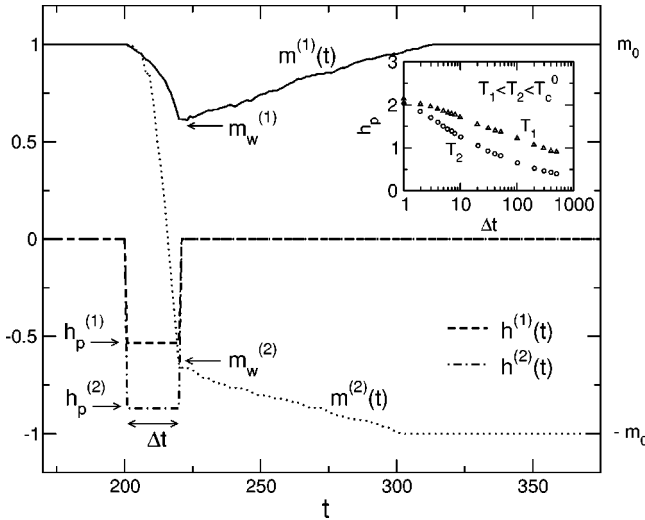
[†]Email address: bikas@cmp.saha.ernet.in

FIG. 1. Typical time variation of the response magnetization $m(t)$ for two different field pulses $h(t)$ with same Δt for an Ising system at a fixed temperature T. The magnetization reversal here occurs due to increased pulse strength, keeping their width Δt same. The transition can also be brought about by increasing Δt, keeping h_p fixed. The inset indicates the typical phase boundaries (where the field withdrawal-time magnetization $m_w = 0$) for two different temperatures (sequential updating; note that for random updating the phase boundaries shift upwards).

fact, a simple application of the domain nucleation theory gives $h_p \ln \Delta t = \text{const}$ along the phase boundary, where the const changes by a factor $1/(d+1)$, where d denotes the lattice dimension, as the boundary changes from single to multidomain region [5].

A mean-field study of the problem gives a qualitative understanding of the diverging time and length scales developed near the transition boundary (in the h_p-Δt plane at a fixed $T < T_c^0$). Mean-field approximation for the dynamics gives the equation of motion for the average magnetization m_i as

$$\frac{dm_i}{dt} = -m_i + \tanh\left(\frac{\sum_j J_{ij} m_j + h(t)}{T}\right). \tag{3}$$

This equation, linearized near the magnetization-reversal transition point, gives, for uniform magnetization,

$$m(t) = \frac{h_p}{\Delta T} - \left(\frac{h_p}{\Delta T} - m_0\right)\left[\exp\left\{\frac{\Delta T}{T}(t - t_0)\right\}\right], \tag{4}$$

as a solution of Eq. (3), for $t_0 \leq t \leq t_0 + \Delta t$. Here $\Delta T = T_c^{mf} - T$, where $T_c^{mf} \equiv J(q=0)$ is the static critical temperature in the mean-field approximation and $J(q)$ is the Fourier transform of the interaction J_{ij}. Due to application of the field h_p, $m(t)$ decreases in magnitude from $m(t_0) \equiv m_0$ to $m(t_0 + \Delta t) \equiv m_w$ at the time of withdrawal of the pulse. Due to absence of fluctuation here, magnetization relaxes back to its original value m_0 if m_w is positive, or to a value $-m_0$ if m_w is negative. In the $t > t_0 + \Delta t$ regime, where $h(t) = 0$, the

magnetization (starting from m_w at $t = t + \Delta t$) relaxes back to its final equilibrium value $\pm m_0$, with a relaxation time [3,5]

$$\tau \sim \frac{1}{(T_c^{mf} - T)} \ln\left(\left|\frac{m_0}{m_w}\right|\right). \tag{5}$$

It diverges at the magnetization-reversal transition boundary, where m_w vanishes. The prefactor gives the divergence of τ at the static mean-field transition temperature, and is responsible for critical slowing down phenomena at the static transition point ($h = 0$). The other factor gives the diverging time scale, at any temperature below the static transition temperature, where magnetization reversal occurs or m_w vanishes due to appropriate combination of h_p and Δt. The solution of the susceptibility $\chi(q)$ gives [3]

$$\chi(q) \sim \exp(-q^2 \xi^2), \tag{6}$$

where the correlation length is given by

$$\xi \sim \left[\frac{1}{(T_c^{mf} - T)} \ln\left(\left|\frac{m_0}{m_w}\right|\right)\right]^{1/2}. \tag{7}$$

Here, too, the prefactor in χ gives the usual divergence at T_c^{mf}, while the other factor gives the divergence at the magnetization-reversal transition point. Incorporating fluctuations, extensive Monte Carlo simulation studies have also convincingly demonstrated [6] that the fluctuation in the order parameter $|m_w|$ and in the internal energy of the system grows with the system size and diverges at the magnetization-reversal transition boundary, where m_w vanishes.

III. MONTE CARLO STUDY AND THE RESULTS

Here, the Monte Carlo study has been carried out in two dimensions (square lattice) with periodic boundary conditions. Spins are updated following Glauber dynamics. The updating rule employed here are both sequential as well as random. In sequential updating rule one Monte Carlo step consist of a complete scan of the lattice in a sequential manner; while in random updating a Monte Carlo step is defined by $N (=L^2)$ random updates on the lattice, where N is the total number of spins in a lattice of linear size L. Studies have been carried out at temperatures below the static critical temperature ($T_c^0 \simeq 2.27$). The system is allowed to evolve from an initial state of perfect order to its equilibrium state at temperature T. The time t_0 is chosen to be much larger than the static relaxation time at that T, so that the system reaches an equilibrium state with magnetization $+m_0(T)$ before the external magnetic field is applied at time $t = t_0$. The field pulse of strength $-h_p$ is applied for duration Δt (measured in Monte Carlo steps or MCS). The magnetization starts decreasing from its equilibrium value m_0. The average value of the magnetization m_w at the time of withdrawal of pulse is noted. The phase boundary of this dynamic transition is defined by appropriate combination of h_p and Δt that produces the magnetization reversal by making $m(t_0 + \Delta t) \equiv m_w = 0$ from a value $m(t_0) = m_0$, i.e., m_w changes sign across the
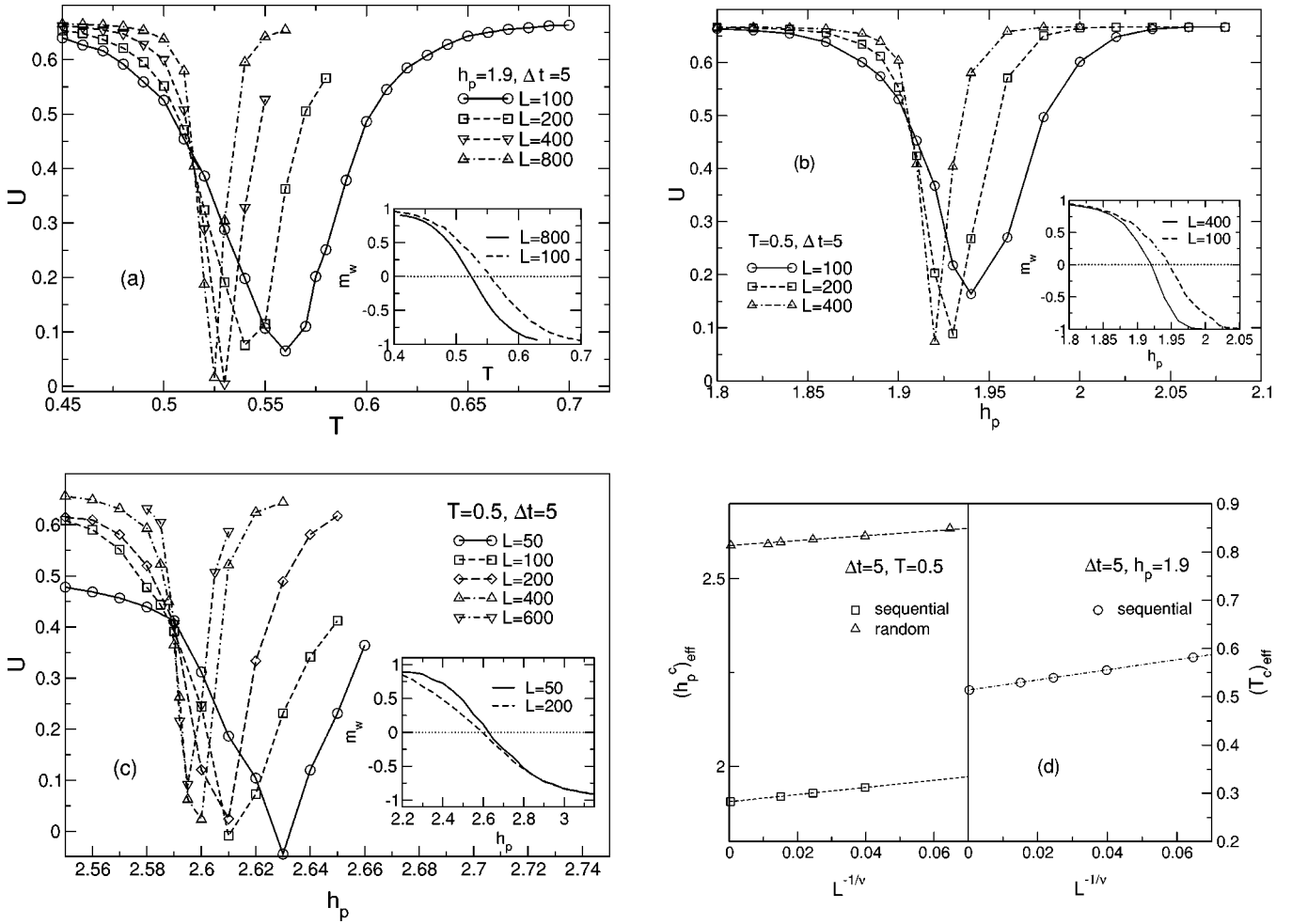
FIG. 2. Behavior of U near the transition, driven by (a) T at a fixed value of h_p ($=1.9$) and Δt ($=5$) with sequential updating, (b) h_p at a fixed value of T ($=0.5$) and Δt ($=5$) with sequential updating, and (c) h_p at a fixed value of T ($=0.5$) and Δt ($=5$) with random updating, for different L, averaged over 1000–20000 initial configurations. The fluctuations are smaller than the symbol size. The insets show the typical behavior of the magnetization m_w at the time of withdrawal of the field pulse by varying (a) T at a fixed h_p and Δt, for $L=100$ and 800, (b) h_p at a fixed T and Δt, for $L=100$ and 400, (c) h_p at a fixed T and Δt, for $L=50$ and 200; $m_w=0$ at the effective transition point. (d) Finite-size scaling study in this parameter range: the effective T_c or h_p^c values (see the insets), where $m_w=0$, are plotted against $L^{-1/\nu}$ with $\nu^{-1}=0.7$. The values of the cumulant crossing points in (a)–(c) are taken to correspond the respective transition points for $L\to\infty$.

phase boundary. The phase boundary changes with T. The behavior of different thermodynamic quantities are studied across the phase boundary. These quantities are averaged over 1000–20000 different initial configurations of the system. The fluctuations over the average value are also noted.

Here, we study the behavior of the reduced fourth-order cumulant U [7] near the magnetization-reversal transition. This is defined as

$$U = 1 - \frac{\langle m_w^4 \rangle}{3 \langle m_w^2 \rangle^2}, \qquad (8)$$

where $\langle m_w^4 \rangle$ is the ensemble average of m_w^4. $\langle m_w^2 \rangle$ is similarly defined. The cumulant U here behaves somewhat differently, compared to that in static and other transitions: Deep inside the ordered phase $m_w \simeq 1$ and $U \to 2/3$. For other (say, static) transitions the order parameter (m_w) goes to zero

with a Gaussian fluctuation above the transition point, giving $U \to 0$ there. Here, however, due to the presence of the pulsed field, $|m_w|$ is nonzero on both sides of the magnetization-reversal transition. Hence, U drops to zero at a point near the transition and grows again after it.

The universality class of the dynamic transition in Ising model under oscillating field has been studied extensively by investigating [2] the critical point and the cumulant value U^* at the critical point, where the cumulant curves cross for different system sizes L. In that case, of course, the variation of U (at any fixed L) is similar to that in the static Ising transitions ($U=2/3$ well inside the ordered phase and $U \to 0$ well within the disordered phase). In fact, U^* value in this oscillatory field case was found to be the same as that in the static case, indicating the same universality class [2]. We observe different behavior in the field-pulse-induced magnetization-reversal transition case.
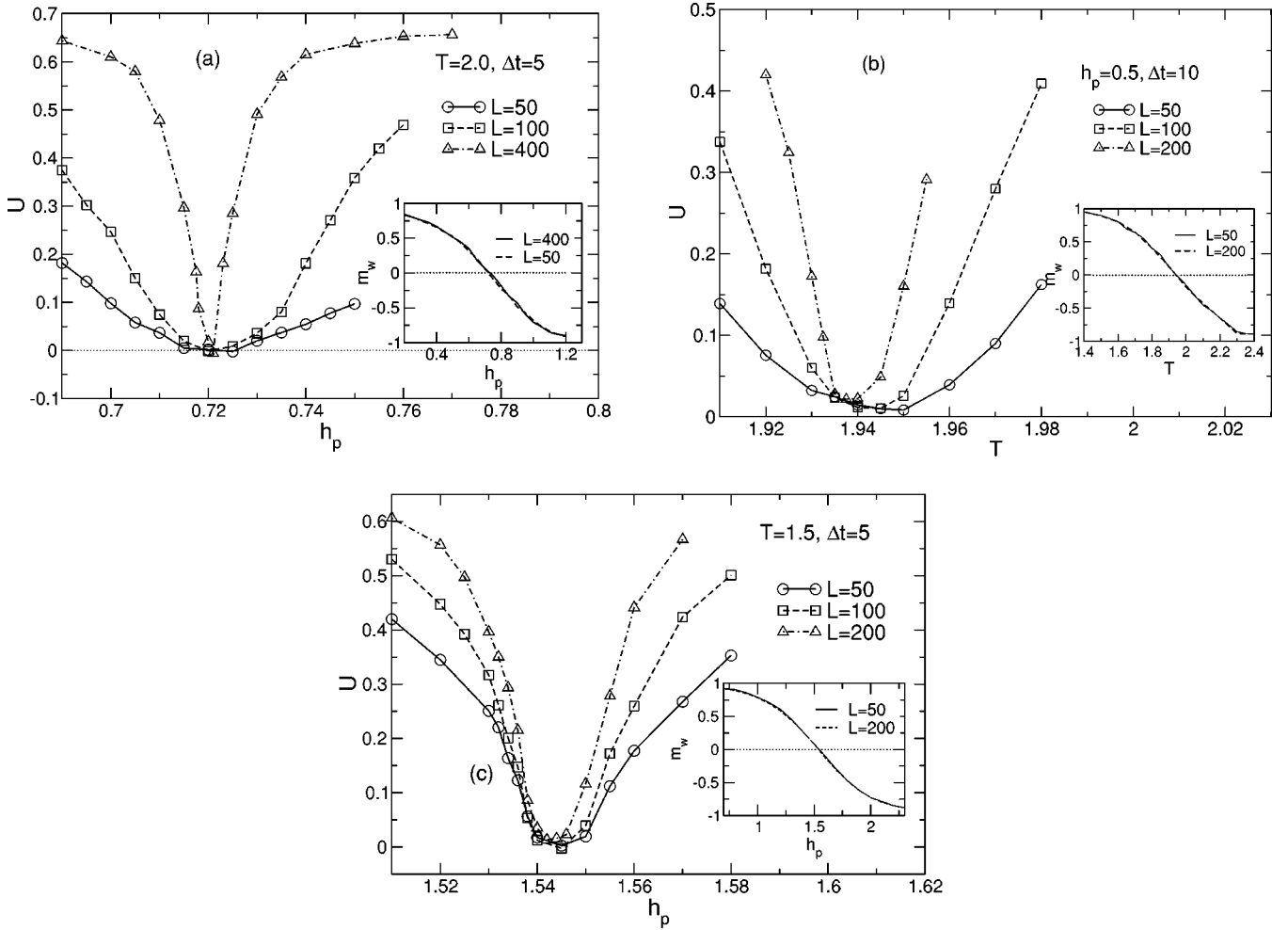
FIG. 3. Behavior of U near the transition, driven by (a) h_p at a fixed value of T ($=2.0$) and Δt ($=5$) with sequential updating, (b) T at a fixed value of h_p ($=0.5$) and Δt ($=10$) with sequential updating, and (c) h_p at a fixed value of T ($=1.5$) and Δt ($=5$) with random updating, for different L, averaged over 1000–6000 initial configurations. The fluctuations are smaller than the symbol size. The insets show the typical behavior of the magnetization m_w at the time of withdrawal of the field pulse by varying (a) h_p at a fixed T and Δt, for $L = 50$ and 400, (b) T at a fixed h_p and Δt, for $L = 50$ and 200, (c) h_p at a fixed T and Δt, for $L = 50$ and 200; $m_w = 0$ at the transition point.

We observe two kinds of distinct behavior of the cumulant U. Typically, for low temperature and low pulse-duration region (see the inset in Fig. 1) of the magnetization-reversal phase boundary, the cumulant crossing for different system sizes L occur at $U^* \simeq 0.42$–0.46 (see Fig. 2). As mentioned already, we have checked these results for both sequential and random updating. Specifically, for $T = 0.5$ and $\Delta t = 5$, [see Fig. 2(c)], we find the transition point value of $h_p \simeq 2.6$, to be smaller than the value ($\simeq 1.9$) for sequential updating. However, the value of U^* at this transition point is again very close to about 0.44. This indicates that updating rule does not affect the universality class (U^* value), as long as the proper region of the phase boundary is considered. For relatively higher temperature and pulse-duration region of the phase boundary, the crossing of U for different L values occur for $U^* \simeq 0_+$. This is true for both sequential [Figs. 3(a)-3(b)] and random [Fig. 3(c)] updating. It may be noted that the phase boundary changes with the updating rule, as the system relaxation time (which matches with the pulse width at the phase boundary) is different for sequential and random updating [7].

It might be noted that in the low temperature and Δt regions, there seems to be significant finite-size scaling of the transition ($m_w = 0$) point [see the insets of Figs. 2(a)-2(c)]. In fact, in Fig. 2(d), the finite-size scaling analysis of those data is presented. For the other cases, there seems to be no significant finite-size effect on the transition point [cf. insets of Figs. 3(a)–3(c)], indicative of a mean-field nature of the transition in this range. It may be noted that to compare the finite-size effects, we normalize the parameters T or h_p by their ranges required for full magnetization reversal. In fact, this weak finite-size effect for high T and Δt regions did not lead to any reasonable value for the fitting exponent in the scaling analysis.

For the static transition of the pure two-dimensional Ising system, $U^* \simeq 0.6107$ [7–9]. For low temperature (and low Δt) regions of the magnetization-reversal phase boundary, the observed values of U^* (in the range 0.42–0.46) are considerably lower than the above mentioned value for the static transition. There is not enough indication of finite-size effect in the U^* value either (cf. Ref. [2]). This suggests a new

universality class in this range. Also, the finite-size scaling study for the effective transition points here [see Fig. 2(d)] gives a correlation length exponent value ($\nu \simeq 1.4$) larger than that of the static transition. For comparatively higher temperatures (and high Δt), the $U^* \simeq 0_+$ at the crossing point. Such small value of the cumulant at the crossing point can hardly be imagined to be a finite-size effect; it seems unlikely that one would get here also the same universality class and U^* value will eventually shoot up to $U^* \simeq 0.44$ (for larger system sizes), as for the other range of the transition. On the other hand, such low value of U^* might indicate a very weak singularity, as indicated by the mean-field calculations [3] mentioned in the Introduction. In fact, even for the static transition, as the dimensionality increases, and the singularity becomes weaker (converging to mean-field exponents) with increasing lattice dimension, the cumulant crossing point U^* decreases ($U^* \simeq 0.61$ in $d=2$ to $U^* \simeq 0.44$ in $d=4$) [9]. We believe the mean-field transition behavior here, as mentioned earlier, is even weaker in this dynamic case as reflected by the value $U^* \simeq 0_+$, corresponding to a logarithmic singularity [as in Eqs. (5) and (7)].

IV. SUMMARY AND CONCLUSIONS

The universality class of the dynamic magnetization-reversal transition, induced by a competing pulse, in an Ising model on a square lattice, below its static ordering temperature, is studied here using Monte Carlo simulations. Both sequential and random updating have been used. The phase boundary at any T ($<T_c^0$) is obtained first in the h_p-Δt plane. They, of course, depend on the updating rule. The phase boundaries obtained compare well with the nucleation theory estimate $h_p \ln \Delta t = $ const along the boundary [5]. The mean-field theory applications [3,5] indicated time and length [Eqs. (5) and (7), respectively] scale divergences at these phase boundaries. Extensive Monte Carlo studies for the fluctuations in the order parameter $|m_w|$ and internal energies, etc., showed prominent divergences along the phase boundaries [6]. Fourth-order cumulant U of the order parameter distribution is studied here for different system sizes (upto $L = 800$) around the phase boundary region. The crossing point of the cumulant (for different system sizes) gives the transition point and the value U^* of the cumulant at the transition point indicates the universality class of the transition. In the low temperature and low pulse width range, the U^* value is found to be around 0.44 [see Figs. 2(a)–2(c)]. The prominent discrepancy with the U^* value ($\simeq 0.61$) for the static transition in the same model in two dimensions indicates a new universality class for this dynamic transition. Indeed, the finite-size scaling analysis [Fig. 2(d)] suggests a different (larger) value of the correlation length exponent also. For comparatively higher temperatures and higher pulse widths, the U^* values are very close to zero [see Figs. 3(a)–3(c)], and the transitions here seem to fall in a mean-field-like weak-singularity universality class similar to that obtained earlier [3], and indicated by Eqs. (5) and (7). Here, the finite-size effects in the order parameter and the transition point are also observed to be comparatively weaker (see insets of Fig. 3).

[1] M. Acharyya and B.K. Chakrabarti, Phys. Rev. B **52**, 6550 (1995); B.K. Chakrabarti and M. Acharyya, Rev. Mod. Phys. **71**, 847 (1999).

[2] G. Korniss, C.J. White, P.A. Rikvold, and M.A. Novotny, Phys. Rev. E **63**, 016120 (2001); G. Korniss, P.A. Rikvold, and M.A. Novotny, *ibid.* **66**, 056127 (2002).

[3] R.B. Stinchcombe, A. Misra, and B.K. Chakrabarti, Phys. Rev. E **59**, R4717 (1999); A. Misra and B.K. Chakrabarti, J. Phys. A **33**, 4249 (2000).

[4] T. Tóme and M.J. de Oliveira, Phys. Rev. A **41**, 4251 (1990).

[5] M. Acharyya, J.K. Bhattacharjee, and B.K. Chakrabarti, Phys. Rev. E **55**, 2392 (1997); A. Misra and B.K. Chakrabarti, Physica A **246**, 510 (1997); Phys. Rev. E **58**, 4277 (1998).

[6] B.K. Chakrabarti and A. Misra, Comput. Phys. Commun. **147**, 120 (2002).

[7] K. Binder and D. Heermann, *Monte Carlo Simulation in Statistical Physics* (Springer, Berlin, 1988).

[8] G. Kamieniarz and H. Blöte, J. Phys. A **26**, 201 (1993).

[9] P. Meyer, Ph.D. thesis, University of Derby, 2000 (unpublished); also at http://hermetic.nofadz.com/compsci/thesis/chap3.htm